Exam. Code : 103201
Subject Code : 1027

B.A./B.Sc. Semester—I
MATHEMATICS
Paper—II
(Calculus & Trigonometry)

Time Allowed—3 Hours] [Maximum Marks—50

Note :— Attempt **FIVE** questions in all selecting at least **TWO** questions from each section.

SECTION—A

I. (a) Give an example to show that the set Q of rationals does not possess the least upper bound property.

(b) If

$$|x-3|<2, \text{ then } \frac{x^2+2x-2}{x+3} \in \left(\frac{1}{8}, \frac{17}{4}\right).$$ 5,5

II. (a) Prove that

$$\lim_{x \to 0} \sin \frac{1}{x} \text{ does not exist}.$$

(b) Show that $f(x)=\frac{1}{x}$ is continuous in (0, 1], but it is not uniformly continuous on (0, 1]. 5,5

III. (a) Differentiate

$$\frac{x\sqrt{x^2-a^2}}{2}-\frac{a^2}{2}\cosh^{-1}\frac{x}{a} \text{ w.r.t. } x$$

(b) If

$$2y = x\left(1+\frac{dy}{dx}\right), \text{ prove that } \frac{d^2y}{dx^2} \text{ is constant}.$$

(c) If

$$y = \left[\log\left(x+\sqrt{x^2+1}\right)\right]^2, \text{ prove that}$$

$$(1+x^2)\; y_{n+2} + (2n+1)\, xy_{n+1} + n^2 y_n = 0.$$

2,3,5

IV. (a) State and prove Taylor's Theorem with Cauchy's form of Remainder.

(b) Prove that

$$f\left(\frac{x^2}{1+x}\right) = f(x) - \frac{x}{1+x} f'(x) + \left(\frac{x}{1+x}\right)^2 \cdot \frac{1}{2!} f''(x)$$

$$-\left(\frac{x}{1+x}\right)^3 \cdot \frac{1}{3!} f'''(x) + - - - \infty.$$

5,5

V. (a) Evaluate

$$\lim_{x\to 0}\left(\frac{1}{x^2}\right)^{\tan x}.$$

(b) Show that the set $\left\{\frac{2+x}{1+x}, x<0 \text{ and } x \neq -1\right\}$ is neither bounded above nor bounded below.

5,5

SECTION—B

VI. (a) If $\cos\alpha + 2\cos\beta + 3\cos\gamma = 0$ and $\sin\alpha + 2\sin\beta + 3\sin\gamma = 0$, prove that $\cos 3\alpha + 8\cos 3\beta + 27\cos 3\gamma = 18\cos(\alpha + \beta + \gamma)$ and $\sin 3\alpha + 8\sin 3\beta + 27\sin 3\gamma = 18\sin(\alpha + \beta + \gamma)$.

(b) Solve the equation $x^{12} - 1 = 0$ and find which of its roots satisfy the equation $x^4 + x^2 + 1 = 0$. 5,5

VII. (a) Prove that $\cos^6\theta \sin^4\theta = 2^{-9}[\cos 10\theta + 2\cos 8\theta - 3\cos 6\theta - 8\cos 4\theta + 2\cos 2\theta + 6]$

(b) If $\cos^{-1}(u + iv) = \alpha + i\beta$, prove that $\cos^2\alpha$ and $\cosh^2\beta$, are the roots of the equation $x^2 - (1 + u^2 + v^2)x + u^2 = 0$. 5,5

VIII. (a) If α and β are imaginary cube roots of unity, then show that

$$\alpha e^{n\alpha} + \beta e^{n\beta} = -e^{\frac{-n}{2}}\left[\cos\frac{\sqrt{3}}{2}n + \sqrt{3}\sin\frac{\sqrt{3}}{2}n\right].$$

(b) If $A + iB = C\tan[x + iy]$, show that

$$\tan 2x = \frac{2CA}{C^2 - A^2 - B^2} \text{ and } \tanh 2y = \frac{2CB}{C^2 + A^2 + B^2}.$$

5,5

IX. (a) If $\tan(\alpha + i\beta) = i$, α and β being real, prove that α is indeterminate and β is infinite.

(b) Sum to n terms the series :

$$\tan^{-1}\frac{1}{3} + \tan^{-1}\frac{1}{7} + \tan^{-1}\frac{1}{13} + \ldots$$

Also deduce the sum to infinite terms. 5,5

X. (a) If

$-(\sqrt{2}-1) < x < \sqrt{2} - 1$, b show that

$$2\left(x - \frac{x^3}{3} + \frac{x^5}{5} - - -\right) = \frac{2x}{1-x^2} - \frac{1}{3}\left(\frac{2x}{1-x^2}\right)^3 + \frac{1}{5}\left(\frac{2x}{1-x^2}\right)^5 - \ldots\ldots$$

(b) State and Prove DE MOIVRE'S Theorem for Rational index. **5,5**